difference between normal dew point and normal boiling point, dimensionless, $T'_b - T_b / T_d - T_b$

θ_{corr} = corrected temperature parameter, $\theta - \Delta\theta$

π = pressure ratio dimensionless, P_c / P'_c

τ = temperature ratio, dimensionless, T_c / T'_c

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Manuscript received August 17, 1959; revision received November 30, 1959; paper accepted December 2, 1959.

Filter Cake Washing Performance

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Differential equations for a filter cake wash cycle are derived on the assumption that the wash liquor executes plug flow in the pores of the cake, with continuous mass transfer between the liquor and a boundary film of filtrate. These equations are solved and the solutions presented as performance charts. The solutions are compared with certain experimental results.

There is considerable literature on filter cake washing (*6, 7, 8, 9*) based on filtration equations such as Kozeny's (*10, 11*). These filtration equations are essentially hydrodynamic equations, based in general on Hagen-Poiseuille flow, requiring the use of some (generally semiempirical) connection between pressure drop and flow rate in order to predict the behavior of a wash cycle.

There is however a different approach to wash analyses which assumes a known flow pattern in the filter cake and writes differential material balances involving empirical mass transfer coefficients. Rhodes (*5*) presented a performance equation of this general type and correlated it with certain data for sodium chloride washing, assuming that the wash liquor was perfectly mixed with the filtrate so that the mass transfer coefficients did not appear explicitly. His performance equation was accordingly a simple exponential decay in wash liquor solute concentration with time.

The present paper presents transport equations of a more realistic sort, assuming plug flow in the pores of the filter cake, with continuous mass exchange between the wash liquor and a boundary film of filtrate. The appropriate differential equations turn out to be a simple form of those studied by Goldstein (*2*) in another connection and have an explicit mathematical solution.

The physical model assumed for the washing is presented below, as are the differential equations. The mathematical solutions of these differential equations are given and compared with some of Rhodes' data. Figures 1 and 2 present these mathematical solutions in a concise graphical form, and so serve as design charts for a wash cycle. Certain extreme cases are discussed.

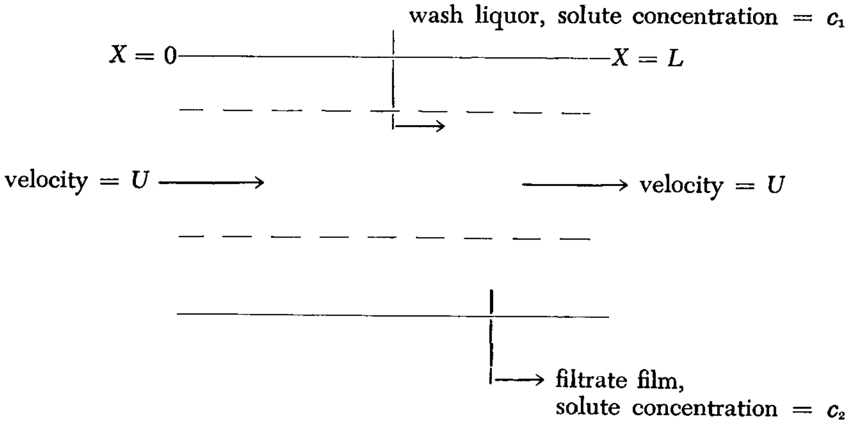
Figures 1 and 2 are plots of the solute concentration in the wash liquor and the filter cake respectively. In making a design to meet certain requirements on the filter cake concentration one would use Figure 2, either at the position corresponding to the highest concentration or at some average position. To make a design in this way requires of course a knowledge of such parameters as mass transfer coefficients which can only be obtained by experiment. In interpreting data, where only exit wash liquor concentrations would generally be available, one would use Figure 1 to determine those values of the parameters which made the theory agree best with the observations.

THE DIFFERENTIAL EQUATIONS

It is assumed that before the start of a wash cycle most of the filtrate has been forced out of the filter cake pore spaces by the pressure difference across the filter cloth. A channel for the flow of wash liquor is thus formed, with a stagnant film of filtrate remaining on its surface. The washing serves to extract the remaining solute from this film.

Whether the pore spaces have been created by the pressure difference across the filter cloth, or whether the excess liquid in the filter cake is pushed out by the first passage of wash liquor, is really of no consequence in what follows. The differential equations below are written only where the wash liquor contacts the pore surface, and the mathematical solutions apply at a given point only after the wash liquor has arrived there.

The flow channel and the filtrate film are taken to be of uniform cross section,



Pore Channel

with the wash liquor executing plug flow down the channel, and with transport of solute between the wash liquor and the filtrate occurring at a rate proportional to the solute concentration difference.

A material balance on solute in a differential slice of the pore gives for the wash liquor

$$\frac{\partial c_1}{\partial t} = h_1(c_2 - c_1) - U\frac{\partial c_1}{\partial x};$$

$$0 < x < L, t > \frac{x}{U} \qquad (1)$$

and for the stagnant film

$$\frac{\partial c_2}{\partial t} = h_2(c_1 - c_2); 0 < x < L, t > \frac{x}{U}$$

To these must be added an inlet condition on the solute concentration in the wash liquor

$$c_1 = \alpha; x = 0, t > 0 \qquad (3)$$

and an initial condition on the solute concentration in the filtrate film

$$c_2 = \beta; Ut < x < L, t > 0 \qquad (4)$$

Equations (1) to (4) represent a complete mathematical specification of the concentration functions c_1, c_2. Their solution will be given below.

MATHEMATICAL SOLUTIONS AND COMPARISON WITH DATA

Equations (1) to (4) are a simple form of those studied by Goldstein (*2*) in connection with fixed bed ion exchange problems. They have the explicit solutions

$$\frac{c_1 - \alpha}{\beta - \alpha} = 1 - e^{-(\xi+\tau)} I_0(2\sqrt{\xi\tau}) - \int_0^\tau e^{-(\xi+\eta)} I_0(2\sqrt{\xi\eta})\,d\eta \qquad (5)$$

$$\frac{c_2 - \alpha}{\beta - \alpha} = 1 - \int_0^\tau e^{-(\xi+\eta)} I_0(2\sqrt{\xi\eta})\,d\eta \qquad (6)$$

In Equations (5) and (6) $I_0(Z)$ is the zero order Bessel function with imaginary argument (*4*):

$$I_0(Z) = J_0(iZ)$$

Numerical values of $I_0(Z)$ and $\int_0^\tau e^{-(\xi+\eta)} I_0(2\sqrt{\xi\eta})\,d\eta$ are tabulated by Jahnke and Emde (*3*) and Brinkley (*1*) respectively.

Figure 1 is a set of plots of the dimensionless solute concentration in the wash liquor (5) against τ for various ξ. Figure 2 is the corresponding set of plots for the dimensionless solute concentration in the filtrate film (*6*).

In comparing the mathematical solutions (5) and (6) with data one must bear in mind that the only quantity generally available to the experimenter is the outlet concentration of solute in the wash liquor. At the pore outlet the dimensionless variables ξ and τ become

$$\xi = \frac{h_1 L}{U} \text{ and } \tau = h_2 t - \frac{h_2}{h_1}\xi$$

The matching of the mathematical solution to data thus becomes a question of finding those values of the parameters

$$\frac{h_1 L}{U}, \frac{h_2}{h_1}, h_2, \alpha, \beta$$

which give the best agreement between the observed values of $c_1(L, t)$ and those computed from (5). It should be noted that α may be assumed known and β often taken equal to zero.

In the experimental data of Rhodes (*5*) runs 1 and 3, for which data are given for the longest times, were taken for numerical analysis with $\alpha = 0$. Satisfactory agreement between computation and experiment was obtained with

Run no.	1	3
Flow rate of wash liquor, $\left(\frac{\text{in.}}{\text{sec.}}\right)$	0.0047	0.0030
Thickness of cake, (in.)	0.5	1.1
$\frac{h_1 L}{U}$	0.5	2.5
h_2/h_1	1.7	0.696
h_2	0.0085	0.005
β	0.015	0.015

The comparison is shown in Figures 3 and 4.

The very satisfactory agreement between the present theory and Rhodes' data shown in Figures 3 and 4 can of course also be obtained by a variety of semiexperimental formulas. Rhodes' formula in particular fits his data very well. These semiempirical formulas however cannot easily account for the sharply dropping concentration profiles

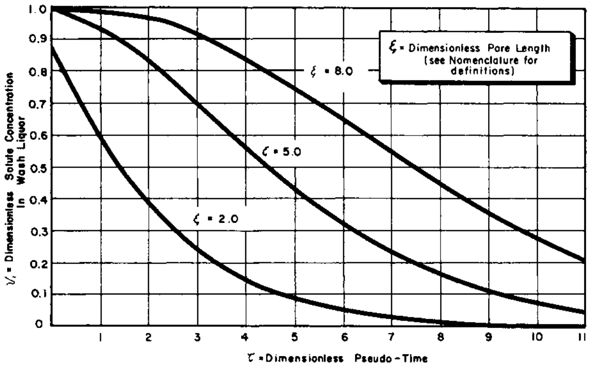


Fig. 1. Solute concentration in wash liquor.

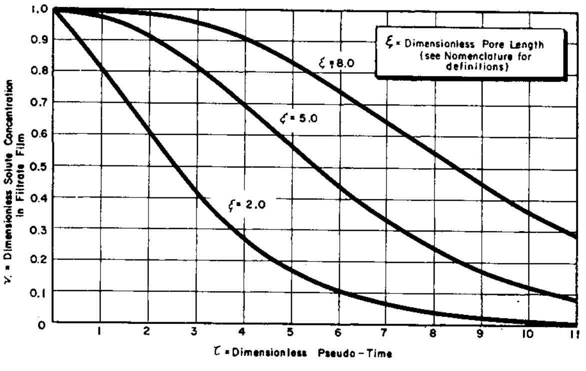


Fig. 2. Solute concentration in filtrate film.

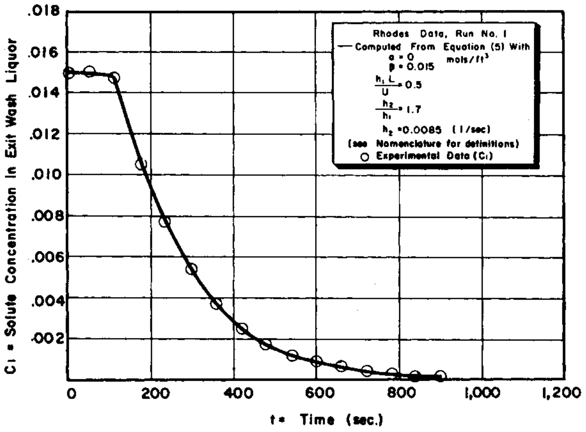


Fig. 3. Solute concentration in exit wash liquor.

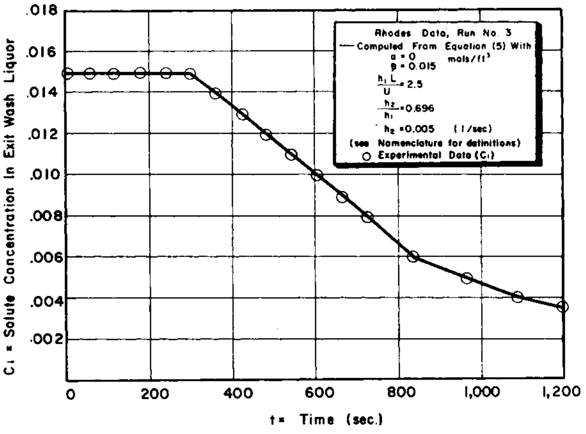


Fig. 4. Solute concentration in exit wash liquor.

appearing in certain modes of operation, where the wash liquor pushes out the solute before it as a piston. The present theory accounts in a straightforward way for their pistonlike behavior by taking h to be large.

EXTREME CASES

An interesting special case of the solutions (5) and (6) arises when h is taken arbitrarily to be large. In this case the system cannot support any concentration difference between the wash liquor and the filtrate film, and the solution becomes, as shown by Goldstein (2)

$$c_1(x, t) = c_2(x, t) = \begin{cases} \alpha \,;\, x < Ut \\ \beta \,;\, x > Ut \end{cases} \tag{7}$$

This solution represents the typical plug displacement of filtrate from the cake pores.

The performance equation given by Rhodes (5) is not a special case of (5) and (6) in the same sense as (7) is. It might be obtained rather by introducing some mixing mechanism into the differential Equations (1) to (4) and then allowing the mixing to become better without bound. The solute concentration in the outlet wash liquor stream, $c_1(L, t)$, would then, by ordinary material balance, satisfy the differential equation

$$\begin{cases} \dfrac{dc_1}{dt} = \dfrac{\alpha - c_1}{\theta} & ;\ t > 0 \\ c_1 = \beta & ;\ t = 0 \end{cases} \tag{8}$$

The parameters α and β have the same meaning as in (1), (2), (3), and (4). The initial value problem (8) has the solution

$$c_1(L, t) = \alpha(1 - e^{-t/\theta}) + \beta e^{-t/\theta}$$

When α equals zero, this solution is reduced to

$$c_1(L, t) = \beta e^{-t/\theta}\,;\ t > 0 \tag{9}$$

Rhodes derives his performance equation by taking the time t in (9) to be measured from some observed break point in the c_1 curve, rather than from the beginning of the wash cycle. That is he replaces (9) by an equation of the form

$$c_1(L, t) = \beta e^{-(t-\Delta)/\theta}\,;\ t > \Delta \tag{10}$$

where β, θ, and Δ appear as parameters to be determined by experiment.

ACKNOWLEDGMENT

The author is indebted to Dr. S. Katz, American Cyanamid Company, for many helpful discussions on various aspects of this problem.

NOTATION

A_1 = cross-sectional area of a channel perpendicular to direction of flow, sq. ft.
A_2 = cross-sectional area of film in a channel perpendicular to direction of wash liquor flow, sq. ft.
$c_1(x, t)$ = concentration of solute in wash liquid, mole/cu. ft.
$c_2(x, t)$ = concentration of solute in stagnant film, mole/cu. ft.
h = mass transfer coefficient, mole/sq. ft.-sec.-mole/cu. ft.
h_1 = hS/A_1 = modified mass transfer coefficients, 1/sec.
h_2 = hS/A_2 = modified mass transfer coefficients, 1/sec.
L = length of pore channel, ft.
S = surface area in a channel for mass transfer, sq. ft./ft.
t = time from beginning of wash cycle, sec.
T = over-all washing time, sec.
U = linear velocity of wash liquid, ft./sec.
x = distance from wash-liquid inlet in pore channel, ft.

Greek Letters

α = inlet concentration of solute in wash liquid, mole/cu. ft.
β = initial concentration of solute in stagnant film, mole/cu. ft.
Δ = time lag, sec.
θ = L/U = wash liquor residence time, sec.
ν_1 = $c_1 - \alpha/\beta - \alpha$ = dimensionless concentration of solute in wash liquor
ν_2 = $c_2 - \alpha/\beta - \alpha$ = dimensionless concentration of solute in filtrate film
ξ = $h_1 x/U$ = dimensionless distance from pore inlet
τ = $h_2(t - x/U)$ = dimensionless time from arrival of wash liquor at pore distance X

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Manuscript received April 23, 1959; revision received January 25, 1960; paper accepted January 27, 1960.